Mathematical Analysis - List 4

1. Convert from degrees to radians.

a) $10°$; b) $24°$; c) $45°$; d) $135°$; e) $350°$; f) $1080°$.

2. Convert from radians to degrees.

a) 1; b) $\frac{\pi}{24}$; c) $\frac{7\pi}{12}$; d) $\frac{4\pi}{3}$; e) $\frac{35}{36}\pi$; f) $\frac{21\pi}{12}$.

3. The standard position of an angle occurs when we place its vertex at the origin of a coordinate system and its initial side on the positive x-axis. A positive (`negative`) angle is obtained by rotating the initial side counterclockwise (`clockwise`) until it coincides with the terminal side. Draw, in standard position, the angle whose measure is given.

a) $\frac{\pi}{8}$; b) $120°$; c) $-\frac{\pi}{5}$; d) $-270°$; e) $\frac{7\pi}{4}$; f) $-\frac{7\pi}{3}$.

4. Express in terms of a trigonometric function of $\alpha \in \left(0, \frac{\pi}{2}\right)$:

a) $\sin\left(\frac{3\pi}{2} - \alpha\right)$; b) $\cos\left(\frac{5\pi}{2} + \alpha\right)$; c) $\tan(\pi - \alpha)$; d) $\cot\left(\frac{\pi}{2} + \alpha\right)$.

5. Express in terms of a trigonometric function of an angle from the first quadrant.

a) $\sin\left(-\frac{\pi}{3}\right)$; b) $\cos\frac{9}{2}\pi$; c) $\tan\left(-\frac{95}{3}\pi\right)$; d) $\cot\frac{14}{9}\pi$.

6. Find the exact value of each expression.

a) $\cos\left(-\frac{19}{6}\pi\right) + \cos\frac{5\pi}{6}$; b) $\cos\left(-\frac{21}{4}\pi\right) - \sin\left(-\frac{13\pi}{4}\right)$;

c) $\tan\left(-\frac{7}{3}\pi\right) - \cot\left(-\frac{5}{3}\pi\right)$; d) $\cot\frac{13}{6}\pi + \cot\left(-\frac{17}{6}\pi\right)$.

7. Find the remaining trigonometric ratios.

a) $\sin\theta = \frac{4}{5}, \quad \frac{\pi}{2} < \theta < \pi$; b) $\tan\theta = -7, \quad \frac{3\pi}{2} < \theta < 2\pi$.

8. Prove the identity.

a) $\frac{1+\tan\alpha}{1+\cot\alpha} = \tan\alpha$; b) $\sin^4\alpha + \cos^4\alpha = 1 - \frac{1}{2}\sin^2 2\alpha$; c) $\tan\alpha + \cot\alpha = \frac{2}{\sin 2\alpha}$;

d) $\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$; e) $\sin^4\alpha - \cos^4\alpha = \sin^2\alpha - \cos^2\alpha$; f) $\frac{1}{\cos\alpha} - \cos\alpha = \sin\alpha\tan\alpha$.

g) $\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{\tan^2\frac{\alpha}{2}+1}$; h) $\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$; i) $\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$;

9. Sketch the graph of the function.

a) $y = \cos 2\left(x - \frac{\pi}{4}\right)$; b) $y = \sin x - \left|\frac{1}{2}\sin x\right|$; c) $y = 1 + \cot\left(x + \frac{\pi}{4}\right)$;

d) $y = \tan x + |\tan x|$; e) $y = \sin x + \cos x$; f) $y = |\tan x|\cot x$.

10. Find all values of x that satisfy the equation.

a) $\sin^2 x + \cos x \sin x = 0$; b) $\sin x - 2 = \cos 2x$; c) $\tan^2 x - 2\tan x + 1 = 0$;

d) $\tan x + \tan 2x = \tan 3x$; e) $\sin \sqrt{x} = 0$; f) $\cos \dfrac{1}{x} = 1$.

11. Find all values of x that satisfy the inequality.

a) $2 \sin \left(\dfrac{\pi}{3} - x \right) \geqslant \sqrt{3}$; b) $2 \cos \left(\dfrac{x}{2} - \dfrac{\pi}{6} \right) < -1$;

c) $\tan \left(\dfrac{x}{4} + \dfrac{\pi}{3} \right) > -1$; d) $\sqrt{3} \cot \left(2x + \dfrac{\pi}{4} \right) \leqslant 1$.

12. Find all values of x that satisfy the inequality.

a) $\cos x \leqslant \sin \dfrac{x}{2}$, $x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2} \right]$; b) $\cos x + \sin x \geqslant \sqrt{\dfrac{3}{2}}$;

c) $\cot x - \dfrac{1}{\cot x} < 0$; d) $\tan x \tan 2x \leqslant 1$, $x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2} \right)$.

13. Find the exact value of each expression.

a) $\operatorname{arc\,sin}(0.5)$; b) $\sin \operatorname{arc\,sin}(0.7)$; c) $\arctan \sqrt{3}$;

d) $\operatorname{arc\,sin} \left(\sin \dfrac{5\pi}{4} \right)$; e) $\cos(\arctan 2)$; f) $\sin \left(2 \operatorname{arc\,sin} \left(\dfrac{4}{5} \right) \right)$.

14. Find the domain and range of the function

$$g(x) = \operatorname{arc\,sin}(3x + 1).$$

15. Prove that

$$\cos(\operatorname{arc\,sin} x) = \sqrt{1 - x^2}.$$